JOURNAL OF AIRCRAFT
Vol. 47, No. 6, November–December 2010

Optimal Design of Unitized Structures Using Response Surface Approaches

Sameer B. Mulani,* Pankaj Joshi,† Jing Li,‡ Rakesh K. Kapania,§ and Yung Seok Shin¶
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

DOI: 10.2514/1.47411

Developments in rapid manufacturing techniques such as electron beam freeform fabrication have made it easier to manufacture objects with complex shapes (like panel with curvilinear stiffeners). In this paper, design optimization of stiffened panels is carried out using different types of response surfaces. Placement and sizing variables are optimized using integrated high-performance, commercially available software. The proposed approach has two optimization loops. The sizing optimization is carried out in the first loop through finite element analysis using gradient-based optimization method for different placement variables. In the second loop, optimal placement of stiffeners is carried out using response surfaces. Because of response surface construction using whole design space, the proposed approach does not suffer from getting trapped in a local minima. The proposed approach is applied to square panels subjected to in-plane biaxial and uniform surface pressure loadings. Panel weight is minimized with buckling, displacements, and stress constraints. An approach to perform both placement and sizing optimization of panels with curvilinear stiffeners is also discussed and a surrogate model is developed for placement and shape optimization of stiffeners.

Nomenclature

a = length of the stiffened plate
b = width of the stiffened plate
c = stiffener shape design variable
d = direction vector perpendicular to stiffener reference curve
h = height of the stiffener
t = thickness of the plate
w = width of the stiffener
w_i = weights of the control point P_i of nonuniform rational basis-splines curve
$w_{i,j}$ = weights of the control point $P_{i,j}$ of nonuniform rational basis-splines surface
y_1 = right end y coordinate of the first stiffener
y_2 = left end y coordinate of the first stiffener
y_3 = right end y coordinate of the second stiffener
y_4 = left end y coordinate of the second stiffener
$C(t)$ = nonuniform rational basis-splines parametric curve
L = length of the stiffener
$N_{i,p}$ = B-spline basis function
N_{st} = number of the stiffeners
P_i = control point's coordinates of nonuniform rational basis-splines curve
$P_{i,j}$ = control point's coordinates of nonuniform rational basis-splines surface
$S(u, v)$ = nonuniform rational basis-splines parametric surface
W = mass of the stiffened panel expressed in terms of y_1, y_2, y_3 and y_4, kg
W_1 = mass of the stiffened panel in terms of y_1, kg
W_2 = mass of the stiffened panel in terms of y_2, kg
W_3 = mass of the stiffened panel in terms of y_3, kg
W_4 = mass of the stiffened panel in terms of y_4, kg
W_α = mass of the stiffened panel expressed in terms of y_1, y_2, y_3, y_4 and α, kg
α = shape design parameter for the curvilinear stiffeners
λ = buckling eigenvalue
ρ = density of the material of the stiffened plate
σ = von Mises stress in the stiffened plate

I. Introduction

THE ongoing revolution in computer-aided manufacturing has significantly removed the constraints being placed on the design and manufacturing of a structure due to manufacturing limitations. Availability of high-performance computing and commercially available software such as MSC.NASTRAN [1,2] (a general purpose finite element program for structural response analysis and design optimization) and MATLAB (a versatile tool to perform computations using linear algebra, computational techniques, optimization methods, etc.) is having an enormous effect on the design of structures.

Together, the software for design and the computer-aided manufacturing are bound to play a major role in the design of all future aerospace and other related structures. We envision an environment in which design and manufacturing, using modern information technology, would be integrated into one step, this is also consistent with the modern trend of employing unitized parts for flight vehicle structures [3]. The integrated capability of computer-aided design and engineering implemented in the MATLAB environment to provide a methodology and a practical design toolbox, *EBF3Paneloptimization*, is shown in the work by Kapania et al. [4] for the optimization of unitized panels with stiffeners. In that work of Kapania et al. [4], the sizing optimization was carried for the fixed stiffener configurations and the conclusion was that EBF3Paneloptimization needed optimization technique that can perform both placement and sizing optimization of the stiffened panels.

Global optimization of complex engineering problems using numerical methods like the finite element method (FEM), com-

Presented as Paper 2007-1901 at the 48th AIAA/ASME/ASCHE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Honolulu, HI, 23–27 April 2007; received 28 September 2009; revision received 1 July 2010; accepted for publication 1 August 2010.

*Postdoctoral Research Associate, Department of Aerospace and Ocean Engineering. Member AIAA.

†Graduate Research Assistant, Department of Aerospace and Ocean Engineering. Member AIAA.

‡Former Graduate Research Assistant, Department of Aerospace and Ocean Engineering. Member AIAA.

§Mitchell Professor, Department of Aerospace and Ocean Engineering. Associate Fellow AIAA.

¶Former Visiting Professor, The Charles E. Via, Jr. Department of Civil and Environmental Engineering. Member AIAA.

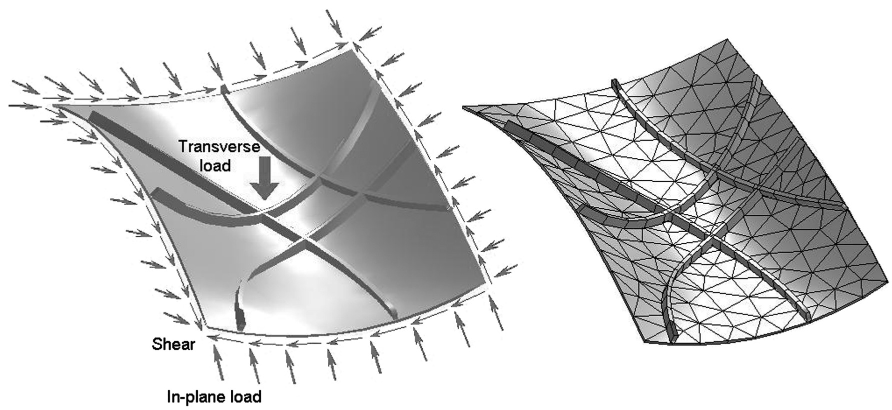

Fig. 1 A curved panel with arbitrarily oriented stiffeners and its finite element mesh.

putational fluid dynamics (CFD), or boundary element methods (BEM) soon become computationally intensive and prohibitive to carry out, so analytical definitions of these complex systems are created using metamodeling techniques [5] such as response surfaces (RS) using polynomials [6–9], krigging or neural networks (NN) by performing complex analyses at few locations. Once the RSs or metamodels are created, the optimization can be carried out using gradient-based techniques or global optimization techniques. During the construction of RSs, two opposing aims, computational efficiency and accuracy, become important factors.

RS construction has the curse of dimensionality meaning that as the number of design variables increases, the RS loses its efficiency over actual complex system evaluation, so different techniques are used to find out important design variables of the problem [10]. Instead of using a global RS, an RS can be constructed over a smaller portion of the whole design space during the optimization process and this local RS is moved over the design space as the optimization process takes its own course [11]. Many times, gradient definitions are included to improve the definition of krigging models and it is called as cokrigging [12]. In this paper, our efforts have been to divide design variables and construct RSs for different levels and make optimization efficient for the problem at hand meaning that a multistep optimization algorithm is employed.

The design optimization of square, stiffened panels is attempted using RS approaches. In the proposed approach, the optimization is carried out in two stages. In the first stage, sizing design variables are optimized for fixed location of the stiffeners using MSC.NASTRAN and, in the second stage, placement optimization is carried out through RS construction. Although our current effort has been almost automated and mainly deals with planar stiffened unitized panels, our final goal is to implement the automatic placement and size optimization of both planar and curvilinear stiffened panel as shown in Fig. 1, and thus provide a capability to design advanced unitized curved panels with curvilinear stiffeners. The work in this paper is an effort to optimize low-cost high-performance panels that have stiffeners with a prescribed uniform or nonuniform pattern, such as isogrid and geodesically stiffened panels [13–15]. Advances in manufacturing and computer-aided design and engineering are making it possible to fabricate these advanced unitized structures.

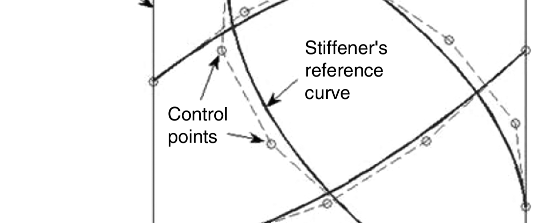

Fig. 2 NURBS representation for stiffeners' reference curves.

In this paper, Sec. II explains the developed design optimization tool called EBF3Paneloptimization. Section III discusses the placement optimization of straight stiffeners using two RS approaches followed by a numerical example in Sec. IV. Section V discusses the placement optimization of curvilinear stiffeners with a fixed curvature followed by placement optimization of curvilinear stiffeners with a variable curvature in Sec. VI. Section VII ends the paper with summary and conclusion.

II. EBF3Paneloptimization Tool

In EBF3Paneloptimization, nonuniform rational basis-splines (NURBS) [16],** DistMesh [17], and MSC.NASTRAN are integrated in a MATLAB environment [4]. Panel and stiffeners's geometries are represented using NURBS. The mesh for the panel is generated using an automatic mesh generation algorithm, DistMesh. An interface between MSC.NASTRAN and MATLAB was developed to automatically transfer the data between the analysis and optimization process. Without this interface, handling the arbitrarily stiffened panel optimization problem at hand would be a hard and time consuming job.

A. Formulation of Optimization of Stiffened Panels

The optimization of the stiffened panels can be briefly formulated as follows: Find **X**, a vector of the design variables, which includes the thickness distribution t of the plate, the cross-sectional dimension distributions (width w and height h) of a blade-type stiffener along the stiffener's axis, and position vector distribution, $\mathbf{c_{st}}$ of a point that represents the stiffener's axis or reference curve. The subscript st indicates the variables related to stiffeners. Minimize the objective function, e.g., weight of the stiffened panel

$$F(\mathbf{X}) = \underbrace{\rho_{\mathbf{plate}} abt}_{\mathbf{mass\ of\ plate}} + \sum_{\mathbf{st}=1}^{N_{\mathbf{st}}} \underbrace{\rho_{\mathbf{st}} L_{\mathbf{st}} h_{\mathbf{st}} w_{\mathbf{st}}}_{\mathbf{mass\ of\ stiffener}} \tag{1}$$

Subjected to the constraints on the predetermined type of responses, e.g., buckling eigenvalue

$$\lambda \geq \lambda_L \tag{2}$$

and von Mises stress constraint on the plate (and stiffeners)

$$\sigma_L \leq \sigma_{\mathbf{von\text{-}mises}} \leq \sigma_U \tag{3}$$

and side constraints on size design variables

$$t_L \leq t \leq t_U; \qquad w_{L_{\mathbf{st}}} \leq w_{\mathbf{st}} \leq w_{U_{\mathbf{st}}}; \qquad h_{L_{\mathbf{st}}} \leq h_{\mathbf{st}} \leq h_{U_{\mathbf{st}}} \tag{4}$$

and on the shape design variables

**Additional data on NURBS Toolbox for SCILAB and MATLAB available online at http://www.aria.uklinux.net/nurbs.php3http://www.aria.uklinux.net/nurbs.php3 [accessed Aug. 2010].

$$\mathbf{c}_{L_{st}} \leq \mathbf{c}_{st} \leq \mathbf{c}_{U_{st}} \tag{5}$$

The shape design variable vector c_{st} contains the points for stiffener's reference curve as shown in Fig. 2. The objective function, constraints and design variables are normalized to avoid the ill-conditioning of the optimization problem formulation.

B. NURBS Representation of the Stiffened Panels

In the EBF3Paneloptimization tool, the stiffener curves and the panel surfaces are represented using NURBS. Mathematically, a NURBS curve (Fig. 2) is represented by

$$\mathbf{C}(t) = \frac{\sum_{i=0}^{n} N_{i,p}(t) w_i \mathbf{P}_i}{\sum_{i=0}^{n} N_{i,p}(t) w_i} \tag{6}$$

and a NURBS surface (Fig. 3) is represented by

$$\mathbf{S}(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) w_{i,j} \mathbf{P}_{i,j}}{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) w_{i,j}} \tag{7}$$

where p and q are the polynomial degrees, $N_{i,p}$ and $N_{j,q}$ are the B-spline basis functions, $\mathbf{P}_{i,j}$ (or P_i) are control points, $w_{i,j}$ are the weights of the control points.

C. Applications of EBF3Paneloptimization

The EBF3Paneloptimization tool [4] has the capability to perform sizing optimization (weight minimization) of unitized structures under buckling and stress constraints for a fixed stiffener configuration. For the work addressed in this paper, EBF3Paneloptimization is applied to different stiffened panels to observe orientation, location, spacing, and/or curvature effects on the optimization. The geometry, load, boundary conditions, material, and initial and bounds on the values of design variables for the stiffened panel are shown in Fig. 4.

The effects of the orientation of the stiffeners, locations, spacing between stiffeners, number of stiffeners and curvature of stiffeners are shown in the Fig. 5, where α is the perpendicular distance of the middle point of the stiffeners from the reference curve. In Figs. 6a, 6b, and 6d, the left and the right pictures show the geometry and the finite element mesh of stiffened panel, respectively. The orientation of the stiffeners plays a major role in reducing the optimal mass of the stiffened panel and it is shown in Figs. 6a and 6b. The stiffeners are aligned in the compression direction due to applied shear as shown in Fig. 6b while for the case shown in Fig. 6a, the stiffeners are not as effective as for the case in Fig. 6b. The increased number of stiffeners also reduces the optimal mass for the same orientation and it is depicted in Figs. 6b and 6c. Comparing Figs. 6a and 6d, it can be noticed that the curvature also plays a very important role in reducing the optimal mass for the same stiffener orientation. While changing the curvature of two stiffeners, using the middle control point, it is found that there are multiple minima in the optimal designs which are shown in Fig. 7. This figure also shows the effects of the number of optimization iterations.

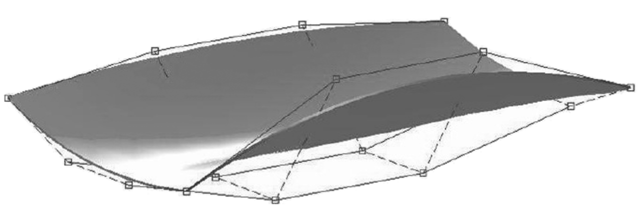

Fig. 3 NURBS representation for panels' reference surface.

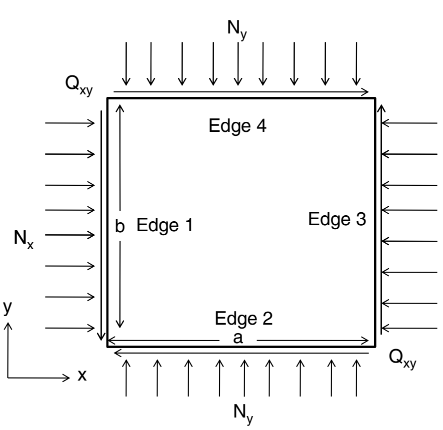

Fig. 4 Geometrical dimensions (a = 2.54 m; b = 2.54 m), pure shear loading ($N_x = 0.0$ kN/m, $N_y = 0.0$ kN/m, and $Q_{xy} = 250$ kN/m), and simply supported conditions of a blade stiffened panel. Both the stiffeners and the plate are made of aluminum. Initial size and bounds; $t_0 = w_0 =$ 0.005 m; $h_0 = 5w_0$; $t_b = w_b = [0.0001, 0.1]$ m; $h_b = [0.0001, 0.5]$ m.

III. Placement Optimization of Straight Stiffeners

Using EBF3Paneloptimization, one can perform the sizing optimization of the stiffened panels easily using gradient-based optimization techniques, however, placement optimization of the stiffeners, especially global optimization, is a difficult task. Therefore, two RS approaches are proposed to do global optimization of stiffened panels. The RS approach is chosen so that new capabilities and new constraints like acoustic performance, and aeroelastic constraints can be integrated easily with EBF3Paneloptimization. Even Pareto optimization can be performed easily for different objectives. In the proposed RS approaches, the surrogate models are developed in terms of the end points of the stiffeners.

A. Constrained Combinatorial Response Surface Approach (CCRSA)

In this approach, a quadratic response surface is fitted in terms of the design variables (the end points of the stiffeners) for which a sizing optimization is subsequently done using MSC.NASTRAN. This approach is explained for the case of the straight stiffeners in the following steps:

1) Distribute the endpoints of the stiffeners which can cover whole area of the plate, e.g., see Fig. 8.

2) Connect each point with the rest of the points, so it is a combinatorial approach.

3) Create a set of designs of different stiffener configurations for the construction of RS.

4) Each end-point pair describes a stiffener.

5) These end-point coordinates are RS variables (called *factors*).

6) Perform optimization using EBF3Paneloptimization for each pair of endpoints.

7) Fit a quadratic surface between response variables and minimize the mass using this obtained RS.

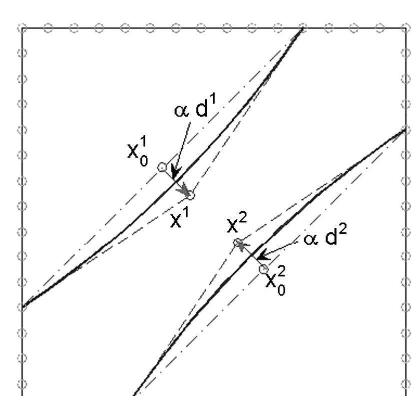

Fig. 5 One shape design parameter, α, controls the motion of two mid points x^1 and x^2 starting from x_0^1 and x_0^2 and by αd^1 and αd^2 for stiffeners 1 and 2, respectively [4].

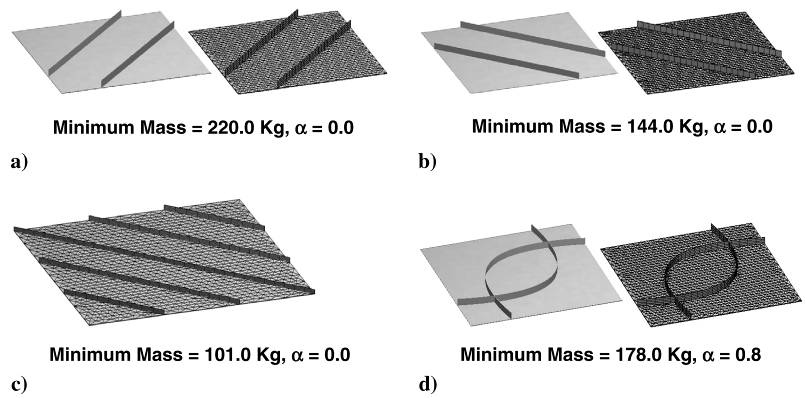

Fig. 6 Effects of the orientation of stiffeners and locations of stiffeners, parts a and b, spacing between stiffeners and number of stiffeners, part c, and curvature of stiffeners, parts a and d, on the optimal designs.

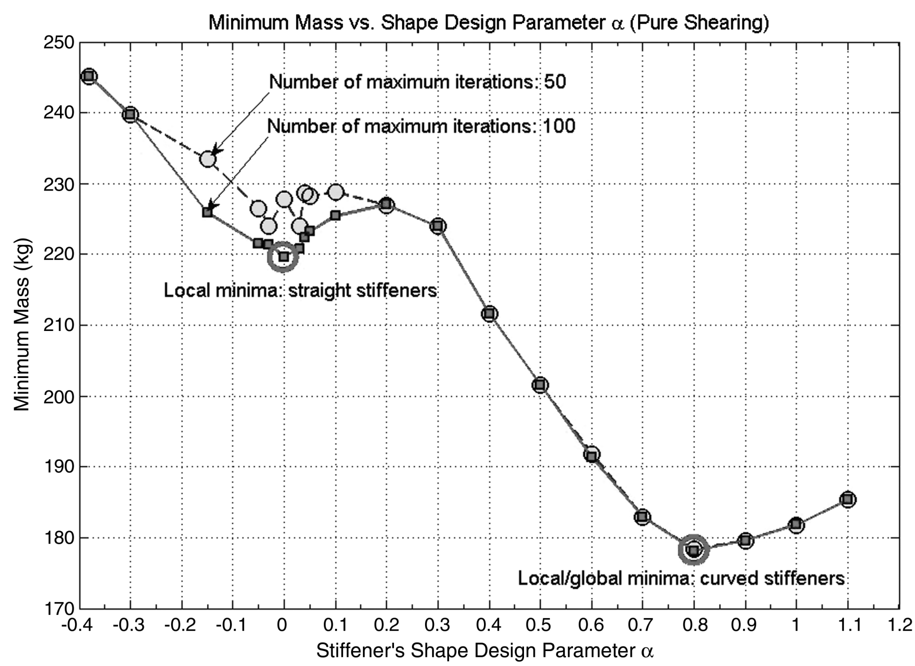

Fig. 7 Minimum mass vs shape design parameter α. One corresponds to straight stiffeners at about $\alpha = 0.0$ with minimum mass $M_{\min} = 220$ kg. The global optimal shape corresponds to curvilinear stiffeners at about $\alpha = 0.8$ with a minimum mass $M_{\min} = 178$ kg.

8) To find the best orientation of the stiffeners, constrain end-point on each edge and try to find the best position on each of the remaining edges using MATLAB optimization subroutines.

B. Constrained Combinatorial Minima + Sensitivity Optimization (CCMSO)

This approach is followed when the CCRSA fails to fit the quadratic RS or there is a significant root mean square error. This approach is carried out in the following manner:

1) Calculate the minimum mass for all different combinations of stiffeners' locations as is done in CCRSA using EBF3Paneloptimization.

2) Create a RS using these combinations and carry out the optimization of placement variables.

3) Using the obtained optimal design variables of the RS, perform an optimization using gradient-based optimization technique and employing sensitivity calculated using the finite difference method.

4) As the RS is created using all possible combinations of the end points of the stiffeners obtained by traversing along all the edges, the whole plate is covered by possible stiffener positions.

The total number of simulations required for the construction of response surface using CCRSA for straight stiffeners is given in Table 1. These approaches can be easily applied to curvilinear stiffeners. The stiffeners are represented using NURBS and the curvature parameters are the control point displacement. The shape of the stiffeners is governed by these control point displacements, which will be additional design variables for the structural optimization discussed in this paper.

IV. Applications of CCRSA and CCMSO

The two approaches, explained in the previous section are applied to the square plate, having boundary conditions defined in Fig. 9. For

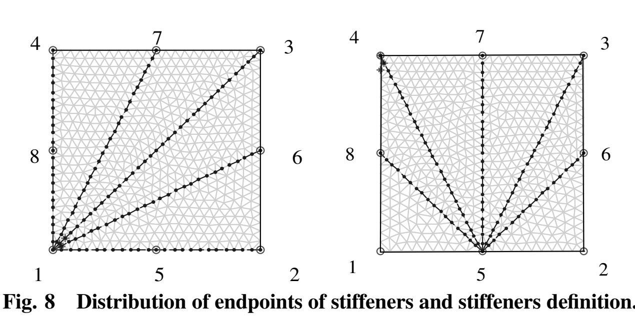

Fig. 8 Distribution of endpoints of stiffeners and stiffeners definition.

Table 1 Number of experiments in CCRSA for different number of straight stiffeners

No. of Stiffeners	No. of Combinations
1	20
2	190
3	1140
4	4845

the loads and boundary conditions shown in Fig. 9, straight stiffeners are studied to find the optimal location of the stiffeners in addition to the sizing optimization. The optimal design of the panels with straight single and double stiffeners are found and these results are shown in Figs. 10 and 11. In Fig. 10, one can see that the optimized mass of the stiffened panel using CCRSA is 273.211 kg and it has only one stiffener. Figure 11 shows the optimal configuration of the stiffeners and their respective size using the CCMSO technique and the optimal mass of the stiffened panel is 223.226 kg, which is 18.295% lower as compared with the mass obtained using the CCRSA approach for a panel with single stiffener. The major compression is applied in the x-direction. Therefore, in both the examples, the stiffeners are aligned in the direction of the major compression load.

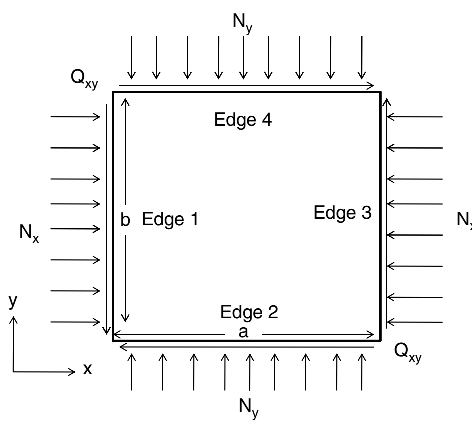

Fig. 9 Biaxial load: $N_x = -250$ kN/m, $N_y = -50$ kN/m, and $Q_{xy} = 0.0$ kN/m. Uniform vertical pressure: $p = 10000$ N/m^2. Fixed: edges 1 and 4. Simply supported: edges 2 and 3. All aluminum $a = b = 2.54$ m, displacement Constraint = ±0.1 m.

V. Placement Optimization of Curvilinear Stiffeners with Fixed Curvature

In Sec. IV, the placement and the sizing optimization of the panel with straight stiffeners was carried out, but the curvature effect of the stiffeners during the optimization was not studied. This section explores the possibility of the placement of the curvilinear stiffeners. Figure 9 gives planar dimensions, load, boundary conditions, and bounds on stiffener size for a square panel with two curvilinear stiffeners. One shape design parameter α, as shown in Fig. 5, is used to control the curvature effect on the two stiffeners.

Response surface methodology (RSM) is used to develop the surrogate model for curvilinear stiffened panel's mass in terms of the curvilinear stiffener's end-point coordinates. A fixed value of shape design parameter $\alpha = 0.1$ is taken to get an idea of the curvature effect. The x coordinate does not change along the left and the right edges of the curvilinear stiffened panel. Therefore, surrogate models are obtained in terms of the y coordinates (y_1, y_2, y_3, and y_4) of stiffener end points. End-point coordinates of curvilinear stiffeners are shown in Fig. 12. Design of experiment software Design-Expert [18] is used to obtain all the RSs. Four RSs, Eqs. (8–11), were generated using central composite design (CCD) in RSM. Numerical values of the coefficients in Eqs. (8–11) are shown in Table 2. In the central composite designs, RS is fitted to a full quadratic model. A central composite design consists of cube points (factorial points) at the corners of a unit cube that is the product of the intervals [−1, 1], star points along the axes at or outside the cube, and center point at the origin. Figure 13 shows a central composite design for two factors (response variables). In the present work, CCD did not give a good quadratic or cubic model. Therefore, four RSs thus obtained are fourth-order polynomials of single variable in the y coordinate of a curvilinear stiffener's end points. It turns out that there are no interaction terms in the model developed by the RSM

$$W_1 = a_1 + a_2 y_1 + a_3 y_1^2 + a_4 y_1^3 + a_5 y_1^4 \tag{8}$$

$$W_2 = b_1 + b_2 y_2 + b_3 y_2^2 + b_4 y_2^3 + b_5 y_2^4 \tag{9}$$

$$W_3 = c_1 + c_2 y_3 + c_3 y_3^2 + c_4 y_3^3 + c_5 y_3^4 \tag{10}$$

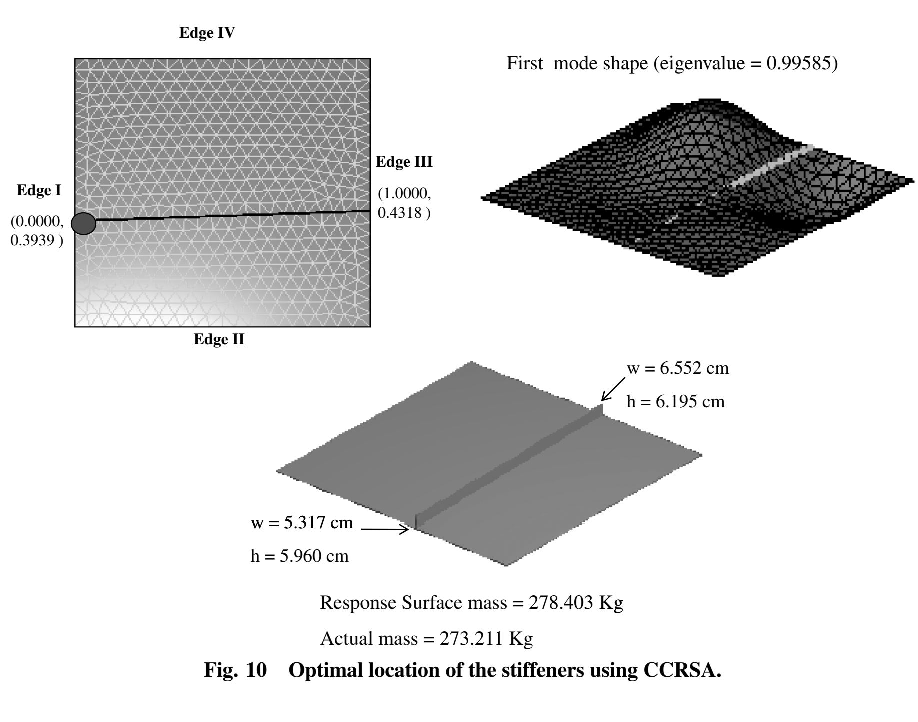

Fig. 10 Optimal location of the stiffeners using CCRSA.

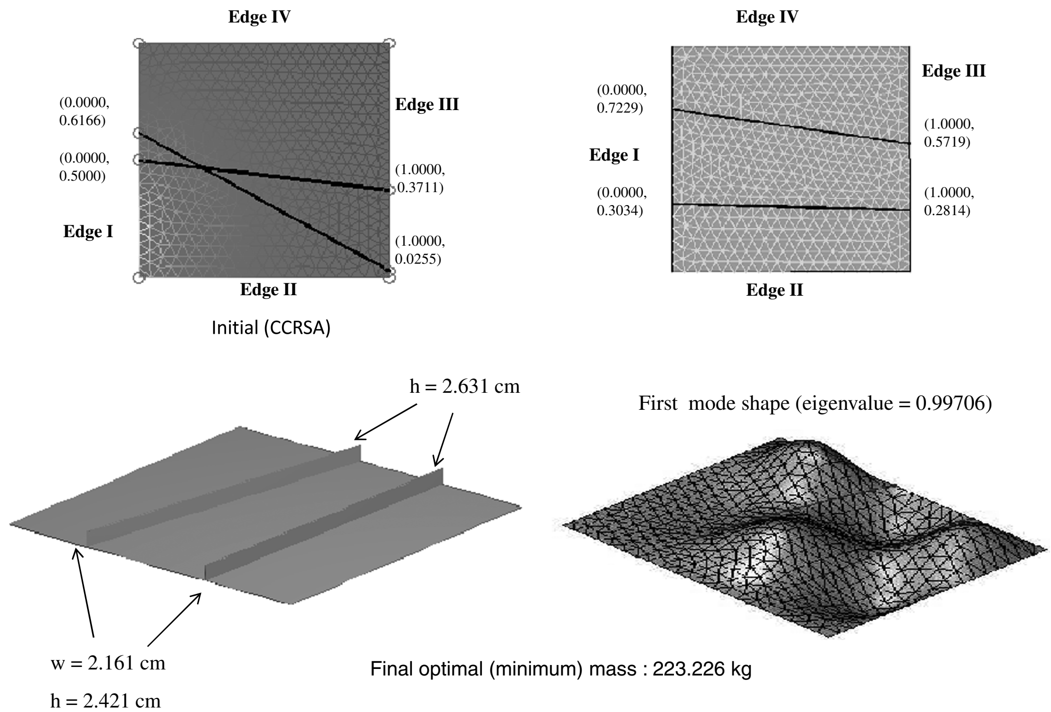

Fig. 11 Optimal location of the stiffeners using CCMSO.

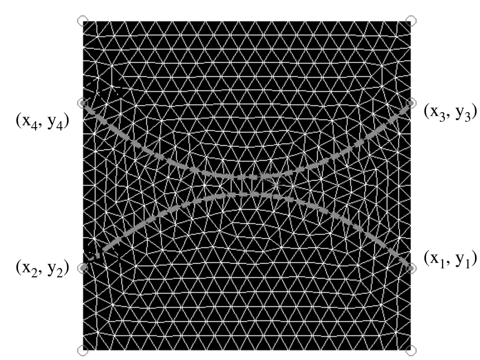

Fig. 12 The end-point coordinates of curvilinear stiffeners on EBF^3 panel.

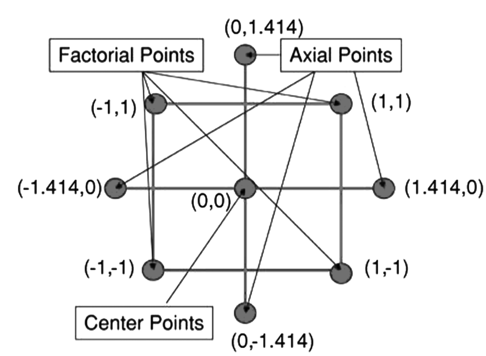

Fig. 13 Central composite design for a two-factor model.

$$W_4 = d_1 + d_2 y_4 + d_3 y_4^2 + d_4 y_4^3 + d_5 y_4^4 \tag{11}$$

To check goodness of fit, other than R^2, both the predicted R^2 and the adjusted R^2 values were obtained. Predicted R^2 gives an indication of the predictive capability of the regression model and adjusted R^2 tells whether the terms used in the surrogate model are significant or not. When R^2 and adj-R^2 differ dramatically, there is a good chance that nonsignificant terms have been included. A very high value of R^2 does not imply a good RS model because, by adding more terms in the RS model, the value of R^2 increases even if the added terms may be insignificant [19]. Numerical values of R^2, the predicted R^2, and, the adjusted R^2 for these four response surfaces are quite high and closely match with each other. These values are shown in Table 3 and show the adequacy of the four response models generated.

Table 2 Numerical values of the coefficients of response surfaces W_1, W_2, W_3, and W_4

Coefficients	Numerical values	Coefficients	Numerical values
a_1	613.304	c_1	353.5
a_2	−2745.277	c_2	−2653.621
a_3	7284.153	c_3	18815.562
a_4	−8314.758	c_4	−54007.697
a_5	3424.027	c_5	54784.437
b_1	261.450	d_1	17292.181
b_2	−574.865	d_2	−92092.159
b_3	2884.043	d_3	185499
b_4	−5381.351	d_4	−165130
b_5	3424.027	d_5	54784.437

From RS W_1, the global minimum value of the mass of the stiffened panel is obtained with the corresponding value of the variable y_1. Similarly, the global minimum values of the mass and corresponding value of y coordinates are obtained from RSs W_2, W_3 and W_4. Values of y_1, y_2, y_3 and y_4 thus obtained from response surfaces W_1, W_2, W_3 and W_4 gave the y coordinates of the end points of two curvilinear stiffeners for the optimum placement of the stiffeners.

Any one of the Eqs. (8–11) can be used to get the mass of the panel with curvilinear stiffeners. Therefore, the final response surface as shown by Eq. (12) is obtained by giving weights a, b, c and d to four response surfaces W_1, W_2, W_3 and W_4, respectively. All four RSs behaved in a similar fashion and gave equal values of R^2, the predicted R^2, and the adjusted R^2 during analysis of variance (ANOVA) test as shown in Table 3. Therefore, equal weights are given to all four RSs. Since each RS was equally efficient in predicting the mass of the stiffened panel, the sum of the weights

Table 3 Results of ANOVA test

RS model	W_1	W_2	W_3	W_4
R-Squared	0.9697	0.9697	0.9697	0.9697
Adj R-Squared	0.9669	0.9669	0.9669	0.9669
Pred R-Squared	0.9633	0.9633	0.9633	0.9633

given to the four RSs is taken to be unity. Figure 14 shows the percentage error in the weight predicted by the RSM model W and it can be seen that the percentage error is less than $\pm 2.0\%$

$$W = aW_1 + bW_2 + cW_3 + dW_4 \tag{12}$$

where $a + b + c + d = 1$.

Figure 15 shows the panel with the optimum placement of two curvilinear stiffeners obtained from four RSM models developed. The final design is shown in Fig. 16. It can be seen that the minimum mass obtained for curvilinear stiffened panel using RSM and EBF3Paneloptimization [4] with a fixed value of shape design parameter ($\alpha = 0.10$) did not improve when compared with a panel with two straight stiffeners. Therefore, as the next step, the effect of variation of shape design parameter, α on the mass of the stiffened panel is presented in the next Section VI.

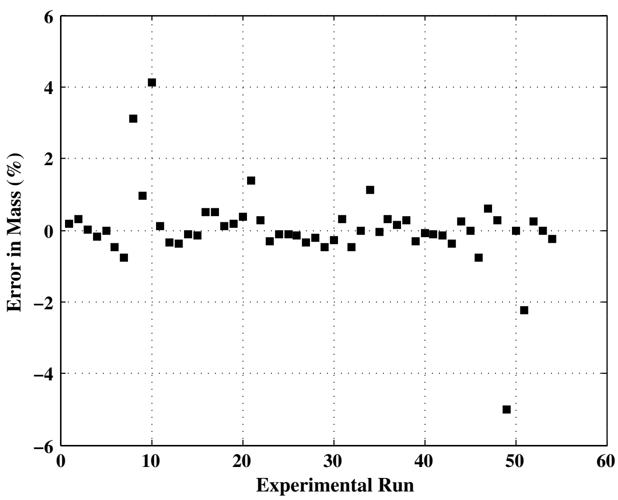

Fig. 14 Percentage error in mass predicted by surrogate model.

VI. Placement Optimization of Curvilinear Stiffeners with Variable Curvature

In the previous section, we developed a surrogate model for a fixed value of the shape design parameter α. This section presents the methodology to develop a surrogate model which incorporates the shape design parameter α and y coordinates of the stiffener end points. Including the shape design parameter α makes the surrogate model development methodology complex and it becomes very hard to choose sample points (also referred to as the points in the design space on which response surface is being fitted) to get a robust surrogate model. To cope with this problem, an approach similar to Queipo et al. [20], see Fig. 17, is followed. In this approach, first the design space is explored for different shape design parameters α. For each value of α, all the possible configurations, as shown in Fig. 18, are searched and a configuration that gives the optimum weight of the panel with curvilinear stiffeners is selected.

This design exploration method gives the knowledge that there is a range of α for which a particular configuration of a curvilinear

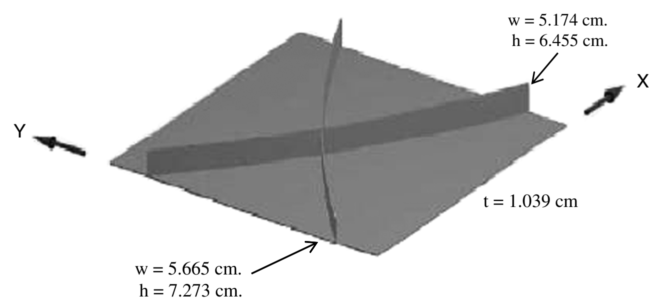

Fig. 16 Final design for shape design parameter $\alpha = 0.1$.

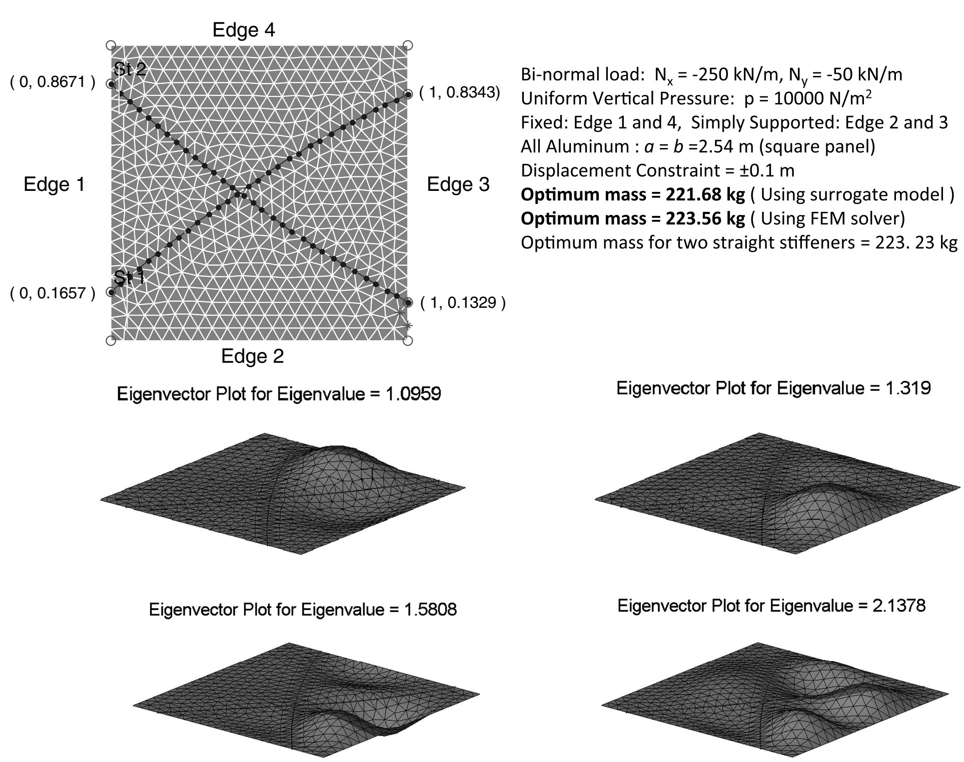

Fig. 15 Optimum design and corresponding mode shape for shape design parameter $\alpha = 0.1$.

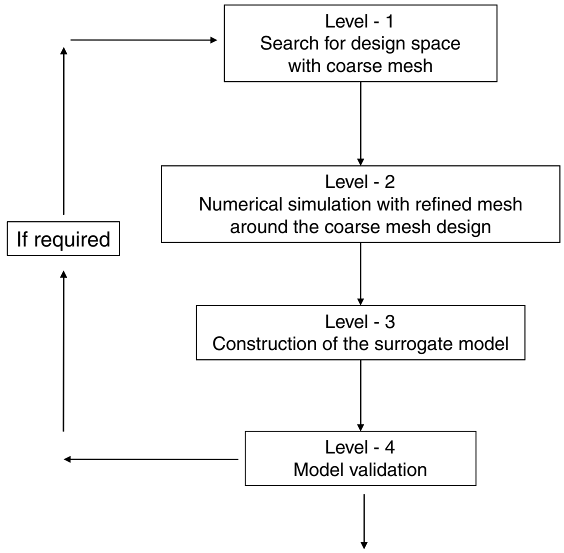

Fig. 17 Different levels to develop a surrogate model.

stiffener gives an optimum design. For the present problem, configuration (a) as shown in Fig. 18, gives the optimum design (minimum mass design) for α varying from 0 to 0.5 To make this process faster, first a coarse mesh for finite element simulation is used so that we have an idea of the design space that contains the optimum design. A refined FEM mesh is used to fit local RS around the optimal design space found using coarse mesh which will predict the mass of the structure within the tolerance specified. If the obtained RS is not robust (does not predict mass within the specified tolerance), level-1 in Fig. 17 is repeated until we get the robust RS. Finally, based on an acceptable error bound in predicting the mass of the panel with curvilinear stiffeners when compared with FEM results, this model is validated.

The surrogate model for the mass of the panels in terms of the end-point coordinates and shape design parameter α was obtained using factorial design in RSM. The end-point coordinates (y_1, y_2, y_3, and y_4) vary from 0 to 1 and α varies from 0.0 to 0.5 and the RS definition is given by Eq. (13). It turns out to be a polynomial of y coordinate of stiffeners end points and shape design parameter α. The ANOVA is performed using the two responses (mass), the actual mass and the mass predicted by the response surface. ANOVA test as shown in Table 4 states that the developed model can be used to predict the mass of the curvilinear stiffened panels thus reducing the computational expense of the design process. The ANOVA test results are close to each other but their individual values are quite low, which suggests that the other metamodeling technique such as kriging, neural networks, etc., may give a better surrogate model. We note that recently, it has been pointed out that for some problems, kriging or NN outperforms RS using lower-order polynomials [21]. From Fig. 19 it can be seen that the percentage error in predicting the mass of the panels is within $\pm 1\%$, which is quite encouraging as far as the tradeoff between computational expense and accuracy is concerned. The developed RS in Eq. (13) is used to check the mass at a point ($y_1 = 0.65$, $y_2 = 0.35$, $y_3 = 0.35$, $y_4 = 0.65$, and $\alpha = 0.45$) in the design space. The mass predicted by developed RS is 214.89 kg. The actual mass is predicted by conducting sizing optimization using MSC.NASTRAN in EBF3Paneloptimization and it turned out to be 210.46 kg. Therefore, it is clear that the panel with curvilinear stiffeners has a lower mass than a panel with two straight stiffeners. This minimum mass design is shown in Fig. 20

Table 4 Results of ANOVA test

R-Squared	0.7835
Adj R-Squared	0.7082
Pred R-Squared	0.5860

$$W_\alpha = 460.3765 - 194.7103\alpha - 218.7234y_1 - 224.7172y_2 + 27.07205y_4 + 250.48431\alpha y_1 - 62.23077y_1y_4 + 347.20690\alpha^2 - 546.45162\alpha^2 y_1 \tag{13}$$

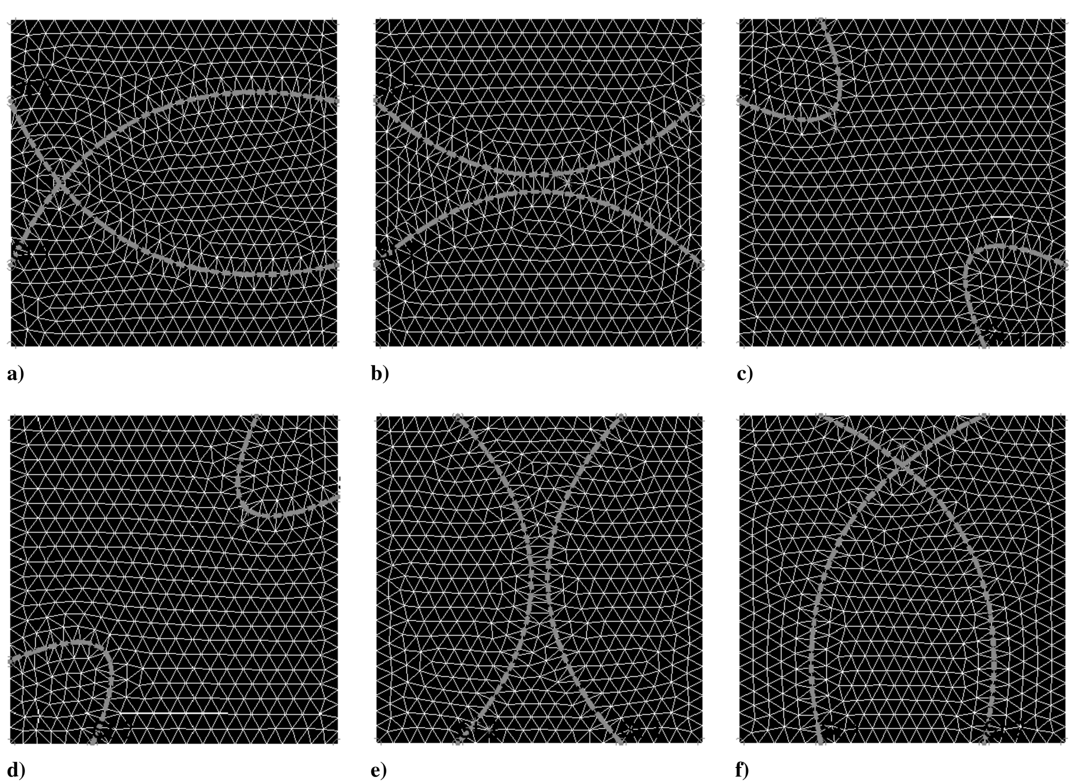

Fig. 18 Possible configurations of panels with two curvilinear stiffeners with $\alpha = 0.45$.

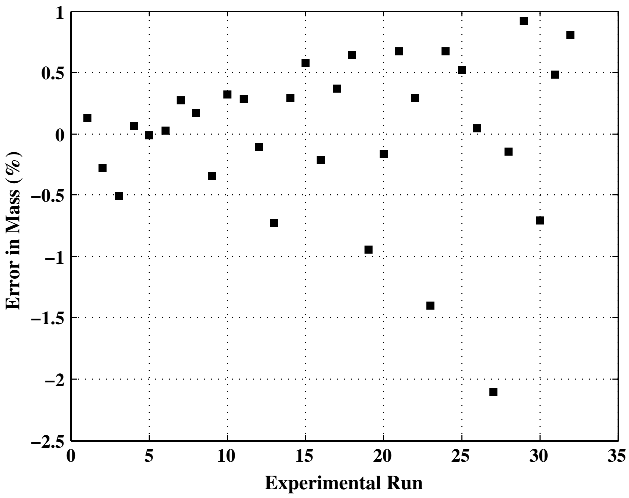

Fig. 19 Percentage error in mass predicted by the surrogate model.

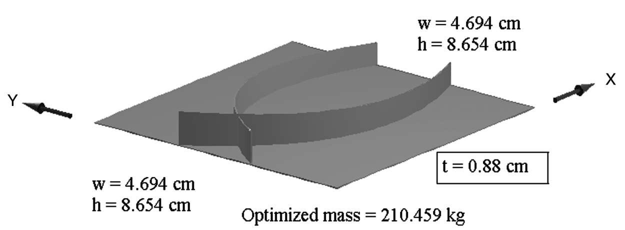

Fig. 20 Minimum mass design for panel with two curvilinear stiffeners.

VII. Conclusions

We have used our integrated code EBF3Paneloptimization to perform global optimization of stiffened panels using different RS approaches. In this work, the placement as well as sizing optimization of straight or curvilinear stiffeners was carried out. To the authors' knowledge, this is the first work to address the placement optimization of stiffeners using RSM. We found that RSM seems to be efficient and that different types of the RSs should be tried to find efficient and accurate RS constructions. To increase the computational efficiency of the optimization, the RS approach should be based on the load sharing capability of the stiffeners for the defined loading conditions. For industrial applications, the development of surrogate models for computationally intensive multidisciplinary design optimization problem is one of the most important tasks as far as feasibility of any design process is concerned. Therefore, we have developed a surrogate model for design optimization of panels with straight or curvilinear stiffeners to perform sizing and shape optimization of a panel with straight or curvilinear stiffeners. For a stiffened panel under biaxial compression with transverse pressure, it is concluded that the panel with two curvilinear stiffeners has a lower mass than a panel with two straight stiffeners. Various other surrogate modeling techniques, namely krigging, neural networks, and radial basis functions are being investigated in our research group to improve both the robustness and the efficiency of the metamodels. To improve the robustness of the metamodels, an ensemble of surrogates and a weighted average surrogate approach is also being considered.

Acknowledgments

The work was conducted as a subcontract from the National Institute of Aerospace with David Peake of the National Institute of Aerospace, Hampton, Virginia, as the grant monitor. The authors thank Karen Taminger of NASA Langley Research Center for substantial technical discussions. Furthermore, we extend our gratitude to the reviewers whose comments were very constructive and helpful in improving this manuscript.

References

[1] MSC.NASTRAN 2005 Quick Reference Guide.

[2] MSC.NASTRAN 2005 Design Sensitivity and Optimization UserUNRECOGNIZED CHARACTER #146s Guide.

[3] Renton, W. J., Olcott, D., Roeseler, W., Batzer, R., Baron, W., and Velicki, A., "Future of Flight Vehicle Structures (2002–2023)," *Journal of Aircraft*, Vol. 41, No. 5, 2004, pp. 986–998. doi:10.2514/1.4039

[4] Kapania, R. K., Li, J., and Kapoor, H., "Optimal Design of Unitized Panels with Curvilinear Stiffeners," *AIAA 5th Aviation, Technology, Integration, and Operations Conference (ATIO)/16th Lighter-than-Air and Balloon Systems Conference*, AIAA, Reston, VA, 2005.

[5] Simpson, T. W., Peplinski, J. D., Koch, P. N., and Allen, J. K., "Metamodels for Computer-based Engineering Design: Survey and Recommendations," *Engineering with Computers*, Vol. 17, No. 2, 2001, pp. 129–150. doi:10.1007/PL00007198

[6] Ganguli, R., "Optimum Design of a Helicopter Rotor for Low Vibration Using Aeroelastic Analysis and Response Surface Methods," *Journal of Sound and Vibration*, Vol. 258, No. 2, 2002, pp. 327–344. doi:10.1006/jsvi.2002.5179

[7] Murugan, S., and Ganguli, R., "Aeroelastic Stability Enhancement and Vibration Suppression in a Composite Helicopter Rotor," *Journal of Aircraft*, Vol. 42, No. 4, 2005, pp. 1013–1024. doi:10.2514/1.5652

[8] Attar, P. J., and Dowell, E. H., "Stochastic Analysis of a Nonlinear Aeroelastic Model Using the Response Surface Method," *Journal of Aircraft*, Vol. 43, No. 4, 2006, pp. 1044–1052. doi:10.2514/1.17525

[9] Viswamurthy, S. R., and Ganguli, R., "Optimal Placement of Piezoelectric Actuated Trailing-Edge Flaps for Helicopter Vibration Control," *Journal of Aircraft*, Vol. 46, No. 1, 2009, pp. 244–253. doi:10.2514/1.38199

[10] Gogu, C., Haftka, R. T., Bapanapalli, S. K., and Sankar, B. V., "Dimensionality Reduction Approach for Response Surface Approximations: Application to Thermal Design," *AIAA Journal*, Vol. 47, No. 7, 2009, pp. 1700–1708. doi:10.2514/1.41414

[11] Vavalle, A., and Qin, N., "Iterative Response Surface Based Optimization Scheme for Transonic Airfoil Design," *Journal of Aircraft*, Vol. 44, No. 2, 2007, pp. 365–376. doi:10.2514/1.19688

[12] Laurenceau, J., Meaux, M., Montagnac, M., and Sagaut, P., "Comparison of Gradient-Based and Gradient-Enhanced Response-Surface-Based Optimizers," *AIAA Journal*, Vol. 48, No. 5, 2010, pp. 981–994. doi:10.2514/1.45331

[13] Gerdon, G., and Gürdal, Z., "Optimal Design of Geodesically Stiffened Composite Cylindrical Shells," *AIAA Journal*, Vol. 23, No. 11, 1985, pp. 1753–1761. doi:10.2514/3.9162

[14] Grall, B., and Gürdal, Z., "Structural Analysis And Design of Geodesically Stiffened Panels With Variable Stiffener Distribution," NASA CR-190608, 1992.

[15] Gürdal, Z., and Gendron, G., "Optimal Design of Geodesically Stiffened Composite Cylindrical Shells," *Composites Engineering*, Vol. 3, No. 12, 1993, pp. 1131–1147. doi:10.1016/0961-9526(93)90070-Z

[16] Farin, G., *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*, 2nd ed., Academic Press, Inc., New York, 1990.

[17] Persson, P.-O., and Strang, G., "A Simple Mesh Generator in MATLAB," *SIAM Review*, Vol. 46, No. 2, 2004, pp. 329–345. doi:10.1137/S0036144503429121

[18] DX, Design-Expert, Software Package, Ver. 7.1 Trial, Stat-Ease, Inc., Minneapolis, MN, 2005.

[19] Myers, R. H., and Montgomery, D. C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiment*, Wiley, New York, 1995.

[20] Queipo, N. V., Haftka, R. T., Shyy, W., Goel, T., Vaidyanathan, R., and Tucker, P. K., "Surrogate-Based Analysis and Optimization," *Progress in Aerospace Sciences*, Vol. 41, No. 1, 2005, pp. 1–28. doi:10.1016/j.paerosci.2005.02.001

[21] Paiva, R. M., Carvalho, A. R. D., Crawford, C., and Suleman, A., "Comparison of Surrogate Models in a Multidisciplinary Optimization Framework for Wing Design," *AIAA Journal*, Vol. 48, No. 5, 2010, pp. 995–1006. doi:10.2514/1.45790